

POTWI 19 SOLUTION

Use inclusion–exclusion on the “bad” paths (those that pass through $(2, 2)$ or $(3, 3)$ or both).

All paths. $\binom{10}{5} = 252$.

Through $(2, 2)$. $(0, 0) \rightarrow (2, 2)$ has $\binom{4}{2} = 6$ paths, and $(2, 2) \rightarrow (5, 5)$ has $\binom{6}{3} = 20$ paths:
 $6 \cdot 20 = 120$.

Through $(3, 3)$. $(0, 0) \rightarrow (3, 3)$ has $\binom{6}{3} = 20$ paths, and $(3, 3) \rightarrow (5, 5)$ has $\binom{4}{2} = 6$ paths:
 $20 \cdot 6 = 120$.

Through both. $(0, 0) \rightarrow (2, 2)$: 6 paths; $(2, 2) \rightarrow (3, 3)$: $\binom{2}{1} = 2$ paths; $(3, 3) \rightarrow (5, 5)$: 6 paths. So
 $6 \cdot 2 \cdot 6 = 72$.

Paths avoiding both.

$$252 - 120 - 120 + 72 = 84.$$

Answer: (A) 84
