

POTWI 18 SOLUTION

Write the two integers as $a = 6m$ and $b = 6n$. Since $\gcd(a, b) = 6$, we need $\gcd(m, n) = 1$. Then

$$\text{lcm}(a, b) = 6mn = 360 \implies mn = 60 = 2^2 \cdot 3 \cdot 5.$$

Because m and n are coprime, each prime power of 60 must go entirely to one of them. The three prime powers 2^2 , 3, 5 can each go to m or to n , giving $2^3 = 8$ ordered pairs (m, n) , hence $8/2 = 4$ unordered pairs:

$$\{m, n\} = \{1, 60\}, \{3, 20\}, \{4, 15\}, \{5, 12\}.$$

Multiplying back by 6, the integer pairs are $(6, 360)$, $(18, 120)$, $(24, 90)$ and $(30, 72)$. Each has $\gcd$ 6 and lcm 360. ✓

Answer: (B) 4
