

POTWI 17 SOLUTION

Let A_3, A_5, A_7 be the sets of integers in 1 to 1000 divisible by 3, 5, 7 respectively. The sizes are

$$\begin{aligned} |A_3| &= \lfloor 1000/3 \rfloor = 333, & |A_5| &= 200, & |A_7| &= 142, \\ |A_3 \cap A_5| &= \lfloor 1000/15 \rfloor = 66, & |A_3 \cap A_7| &= \lfloor 1000/21 \rfloor = 47, & |A_5 \cap A_7| &= \lfloor 1000/35 \rfloor = 28, \\ |A_3 \cap A_5 \cap A_7| &= \lfloor 1000/105 \rfloor = 9. \end{aligned}$$

A number in exactly one set is counted once in $\sum |A_i|$, but a number in exactly two sets is counted twice there and a number in all three is counted three times. The “exactly one” count is therefore

$$\sum |A_i| - 2 \sum |A_i \cap A_j| + 3 |A_3 \cap A_5 \cap A_7|.$$

Substituting:

$$(333 + 200 + 142) - 2(66 + 47 + 28) + 3(9) = 675 - 282 + 27 = 420.$$

Answer: (A) 420
