

POTWI 16 SOLUTION

Factor: $n^3 - n = (n - 1)n(n + 1)$, a product of three consecutive integers. Since $24 = 8 \cdot 3$ with $\gcd(8, 3) = 1$, we need divisibility by both 3 and 8.

Divisibility by 3. Among any three consecutive integers one is a multiple of 3, so this always holds.

Divisibility by 8.

- If n is *odd*, then $n - 1$ and $n + 1$ are consecutive even numbers: one is a multiple of 4 and the other a multiple of 2, so their product is divisible by 8. All 50 odd values of n work.
- If n is *even*, then $n - 1$ and $n + 1$ are odd, so all the factors of 2 come from n itself. We need $8 \mid n$: $n = 8, 16, \dots, 96$, which is 12 values.

Total. $50 + 12 = 62$. **Answer:** (C) 62