

POTWI 13 SOLUTION

Let the circles have centres O_1 (radius 5) and O_2 (radius 8), and let the common external tangent touch them at A and B respectively.

Because the circles are externally tangent, the distance between the centres is

$$O_1O_2 = 5 + 8 = 13.$$

The radii O_1A and O_2B are both perpendicular to the tangent line, so $O_1A \parallel O_2B$ and ABO_2O_1 is a right trapezium. Draw the line through O_1 parallel to AB , meeting O_2B at C . Then O_1ABC is a rectangle, so

$$O_1C = AB, \quad O_2C = 8 - 5 = 3,$$

and triangle O_1CO_2 has a right angle at C with hypotenuse $O_1O_2 = 13$. By Pythagoras,

$$AB^2 = 13^2 - 3^2 = 169 - 9 = 160 \implies AB = \sqrt{160} = 4\sqrt{10}.$$

(General rule: for radii r_1, r_2 with centres d apart, the common external tangent has length $\sqrt{d^2 - (r_1 - r_2)^2}$; for externally tangent circles this equals $2\sqrt{r_1r_2} = 2\sqrt{40}$, the same value.)

Answer: (A) $4\sqrt{10}$