

POTWI 11 SOLUTION

Since $x^2 + y^2 = 1$, the expression becomes

$$x^2 + xy + y^2 = 1 + xy.$$

So we need the maximum of xy . By AM–GM (or from $(x - y)^2 \geq 0$),

$$xy \leq \frac{x^2 + y^2}{2} = \frac{1}{2},$$

with equality exactly when $x = y$, i.e. $x = y = \frac{1}{\sqrt{2}}$ (which does satisfy $x^2 + y^2 = 1$).

Therefore the maximum value is $1 + \frac{1}{2} = \frac{3}{2}$.

(Trigonometric check: with $x = \cos \theta$, $y = \sin \theta$, the expression is $1 + \frac{1}{2} \sin 2\theta$, whose maximum is $\frac{3}{2}$.)

Answer: (C) $\frac{3}{2}$