



Problem of the Week

Problem C and Solution

The Sequence of the Year

Problem

In a sequence of numbers, each number in the sequence is called a *term*. In the sequence 2, 4, 6, 8, the first term is 2, the second term is 4, the third term is 6, and the fourth term is 8.

In another sequence, the first term is 24. We can determine the next terms in the sequence as follows:

- If a term is even, then divide it by 2 to get the next term.
- If a term is odd, then multiply it by 3 and add 1 to get the next term.

By doing this, we can determine that the first three terms in the sequence are 24, 12, and 6.

What is the 2024th term in the sequence?

Solution

We will begin by finding more terms in the sequence. The first 14 terms of the sequence are 24, 12, 6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1.

If we continue the sequence, we will see that the terms 4, 2, and 1 will continue to repeat. Thus, the 9th term, 12th term, 15th term, and so on, will each have a value of 4. Notice that these term numbers are all multiples of 3. It follows that every term number after 9 that is a multiple of 3 will have a value of 4.

Thus, since 2022 is a multiple of 3, the 2022nd term will have a value of 4. Then, the 2023rd term will have a value of 2, and the 2024th term will have a value of 1.

EXTENSION:

In 1937, the mathematician Lothar Collatz wondered if any sequence whose terms after the first are determined in this way would always eventually reach the number 1, regardless of which number you started with. This problem is actually still unsolved today and is called the Collatz Conjecture.