Problem of the Week
Problem D and Solution
Student to Student

Problem

At the beginning of the school year, the ratio of the number of Grade 10 students to the number of Grade 9 students at CEMC H.S. was 15 : 16. By the end of the year, there were 30 more Grade 10 students and there were 20 fewer Grade 9 students, and the ratio of the number of Grade 10 students to the number of Grade 9 students was now 11 : 10.

How many Grade 9 students and how many Grade 10 students were there at the beginning of the school year?

Solution

Originally, the ratio of the number of Grade 10 students to the number of Grade 9 students was 15 : 16. Therefore, we can let the number of Grade 10 students at the beginning of the year be $15n$ and the number of Grade 9 students at the beginning of the year be $16n$, for some integer n.

Thus, at the end of the year, there were $15n + 30$ Grade 10 students and $16n - 20$ Grade 9 students.

Since the ratio of the number of Grade 10 students at the end of the year to the number of Grade 9 students at the end of the year is 11 : 10, we have

$$\begin{aligned} \frac{15n + 30}{16n - 20} &= \frac{11}{10} \\ 150n + 300 &= 176n - 220 \\ 520 &= 26n \\ n &= 20 \end{aligned}$$

Therefore, there were $16n = 16(20) = 320$ Grade 9 students and $15n = 15(20) = 300$ Grade 10 students at the beginning of the school year.